Prosposal for loop phase for the global decoupling

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Experimental observables of global coupling

In the view of instrumentation, the particle's nth turn x and y coordinate can be casted as

$$\left\{ \begin{array}{lcl} x_n & = & A_{1,x}\cos[2\pi Q_1(n-1)+\phi_{1,x}] + A_{2,x}\cos[2\pi Q_2(n-1)+\phi_{2,x}] \\ y_n & = & A_{1,y}\cos[2\pi Q_1(n-1)+\phi_{1,y}] + A_{2,y}\cos[2\pi Q_2(n-1)+\phi_{2,y}] \end{array} \right. , \quad (1)$$

Besides the two eigentunes Q_1 and Q_2, here we define another 4 observable quantities to describe the global coupling.

$$\left\{ \begin{array}{lcl} r_1 & = & |A_{1,y}|/|A_{1,x}| \\ r_2 & = & |A_{2,x}|/|A_{2,y}| \end{array} \right. . \quad (2)$$

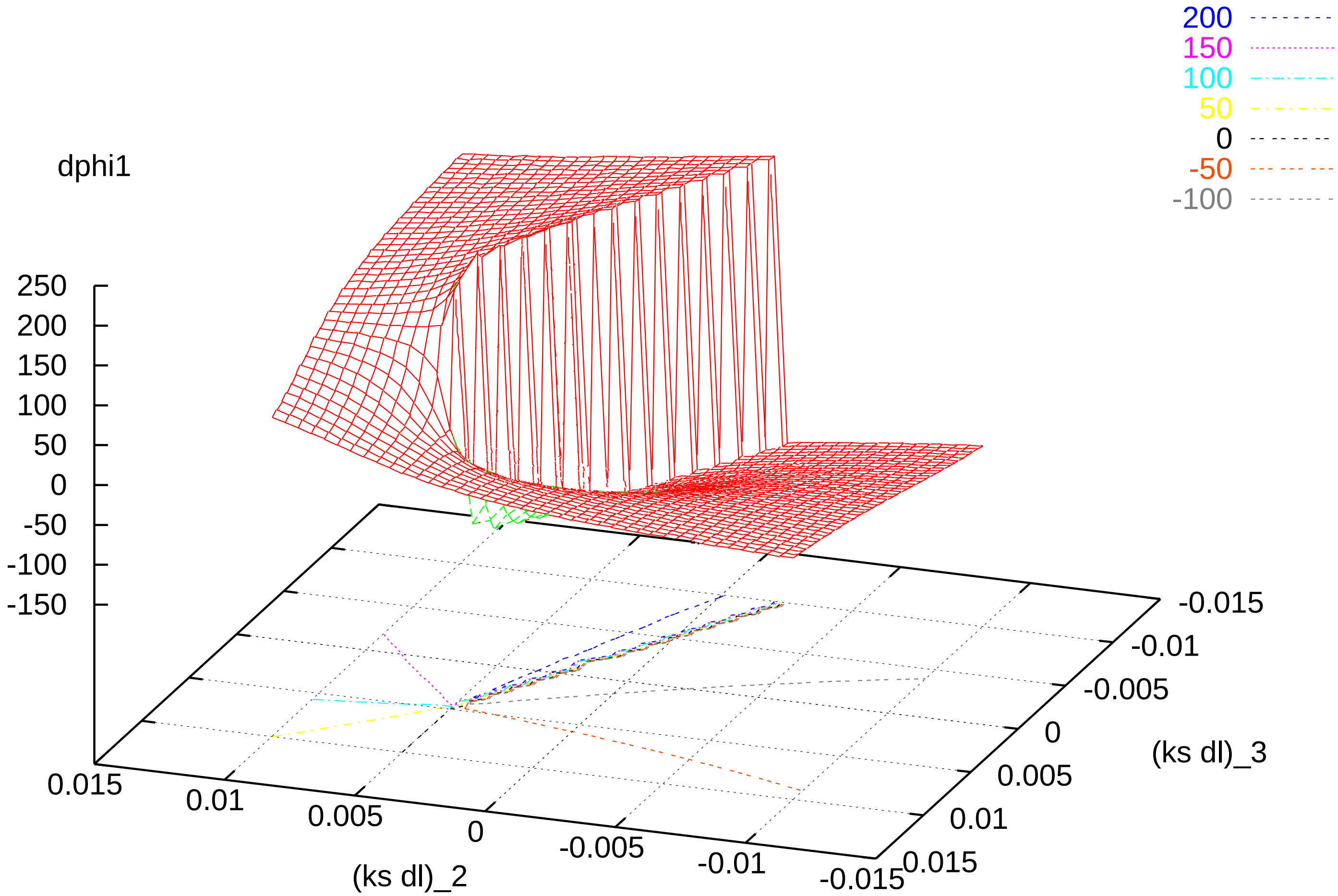
$\Delta\phi_1$ and $\Delta\phi_2$ are the phase differences between the contributions from mode I or mode II into the horizontal and vertical planes,

$$\left\{ \begin{array}{lcl} \Delta\phi_1 & = & \phi_{1,y} - \phi_{1,x} \\ \Delta\phi_2 & = & \phi_{2,x} - \phi_{2,y} \end{array} \right. . \quad (3)$$

You may define your own observables. However, the 6 quantities are good enough. They are measurable from PLL or Turn-by-turn digital BPMs.

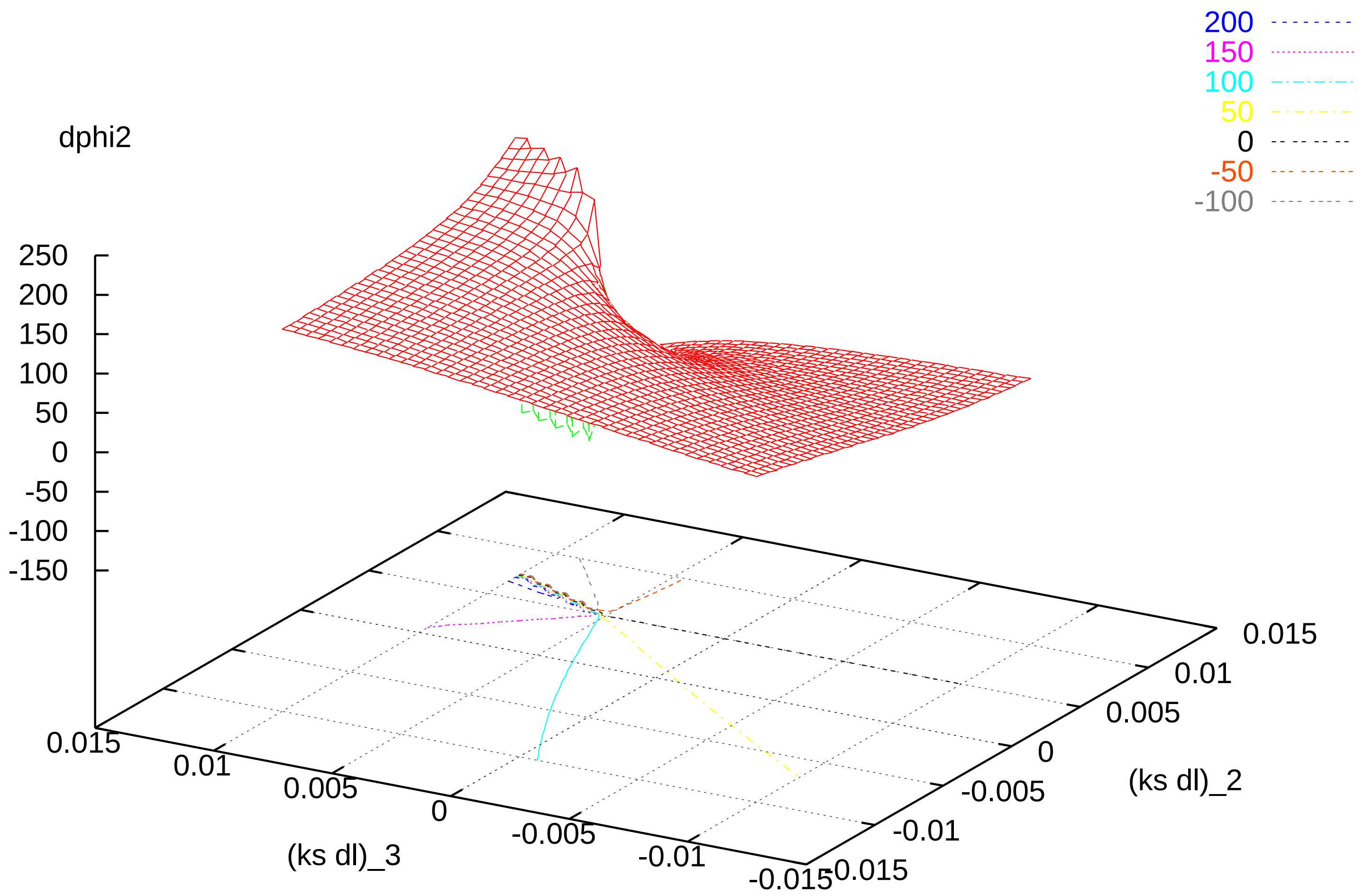
Simulation: 2-D decoupling scan observation

The phase differences $\Delta\phi_1$ during two skew quadrupole scan.



Simulation: 2-D decoupling scan observation

The phase differences $\Delta\phi_2$ during two skew quadrupole scan.



Ananlytical Solution to the 6 quantities

In action-angle parameterization, x and y coordinates are described as

$$\left\{\begin{array}{lcl} x &=& p_{11}\sqrt{2J_1}\cos\Phi_1 + p_{13}\sqrt{2J_2}\cos\Phi_2 - p_{14}\sqrt{2J_2}\sin\Phi_2 \\ y &=& p_{31}\sqrt{2J_1}\cos\Phi_1 - p_{32}\sqrt{2J_1}\sin\Phi_1 + p_{33}\sqrt{2J_2}\cos\Phi_2 \end{array}\right. . \tag{4}$$

It is easy to obtain $r_{1,2}$ and $\Delta\phi_{1,2}$

$$\left\{\begin{array}{lcl} r_1 &=& \sqrt{p_{31}^2 + p_{32}^2}/p_{11} \\ r_2 &=& \sqrt{p_{13}^2 + p_{14}^2}/p_{33} \end{array}\right. , \tag{5}$$

$$\left\{\begin{array}{lcl} \Delta\phi_{1,0} &=& \arctan(p_{32}/p_{31}) \\ \\ \Delta\phi_{2,0} &=& \arctan(p_{14}/p_{13}) \end{array}\right. . \tag{6}$$

or in the Twiss and coupling parameters,

$$\left\{\begin{array}{lcl} r_1 &=& \sqrt{\beta_1 c_{22}^2 + 2\alpha_1 c_{22}c_{12} + \gamma_1 c_{12}} \;/\; (r\sqrt{\beta_1}) \\ \\ r_2 &=& \sqrt{\beta_2 c_{11}^2 - 2\alpha_2 c_{11}c_{12} + \gamma_2 c_{12}} \;/\; (r\sqrt{\beta_2}) \end{array}\right. , \tag{7}$$

$$\left\{\begin{array}{lcl} \Delta\phi_{1,0} &=& \arctan(-c_{12}/(\alpha_1 c_{12} + \beta_1 c_{22})) \\ \\ \Delta\phi_{2,0} &=& \arctan(c_{12}/(-\alpha_2 c_{12} + \beta_2 c_{11})) \end{array}\right. . \tag{8}$$

where we define $\gamma_1 = (1+\alpha_1^2)/\beta_1$, $\gamma_2 = (1+\alpha_2^2)/\beta_2$.

Ananlytical Solution to the 6 quantities

In the frame of Hamiltinian perturbation theory, the particle's x and y coordinates in the two eigenmodes can be written as

$$\left\{\begin{array}{lcl} x(s) & = & \sqrt{2\beta_x}\{a\cos[\Psi_x+(\nu-\Delta/2)\varphi-\chi/2)]+b\cos[\Psi_x-(\nu+\Delta/2)\varphi-\chi/2)]\} \\ y(s) & = & \sqrt{2\beta_y}\{c\cos[\Psi_y+(\nu+\Delta/2)\varphi+\chi/2)]+d\cos[\Psi_y-(\nu-\Delta/2)\varphi+\chi/2)]\} \end{array}\right. \tag{9}$$

There are two eigentunes,

$$\left\{\begin{array}{lcl} Q_1 & = & Q_{x,0}-\frac{1}{2}\Delta+\frac{1}{2}\sqrt{\Delta^2+|C^-|^2} \\ Q_2 & = & Q_{y,0}+\frac{1}{2}\Delta-\frac{1}{2}\sqrt{\Delta^2+|C^-|^2} \end{array}\right. . \tag{10}$$

Therefore the tune split is

$$|Q_1-Q_2|=\sqrt{\Delta^2+|C^-|^2} \quad . \tag{11}$$

$\Delta\phi_{1,2}$ and $r_{1,2}$ are given by

$$\left\{\begin{array}{lcl} r_1 & = & \sqrt{\frac{\beta_y}{\beta_x}}\cdot\frac{|C^-|}{2\nu+\Delta} \\ r_2 & = & \sqrt{\frac{\beta_x}{\beta_y}}\cdot\frac{|C^-|}{2\nu+\Delta} \end{array}\right. , \tag{12}$$

$$\left\{\begin{array}{lcl} \Delta\phi_1 & = & \chi \\ \Delta\phi_2 & = & \pm\pi-\chi \end{array}\right. . \tag{13}$$

Phase Lock Loop for the phase loop

- The above phase loop results based on free oscillation.
- What the definition of the PLL phase difference difinition?
- The difference between the free oscillation and PLL driving oscillation?

Beam experiment plan in coming RHIC run

- To test whether the PLL phase is well defined.

  What's the phase defined in PLL

  What's the resolution of the phase there

- To test the possiblities of the proposed phase loop .

  Introduce the extra coupling into a well decoupled machine

  Decoupling the machine based on the phase loop

- The best way is to do a 2-D scan like the above simulation

  (S. Peggs strongly suggested).